Automata Theory

Alphabet

An alphabet, denoted by Σ , is a finite and nonempty set of symbols.

Example:

1. If Σ is an alphabet containing all the 26 characters used in English language, then Σ is finite and nonempty set, and $\Sigma = \{a, b, c, \ldots, z\}$.
2. $X = \{0,1\}$ is an alphabet.

String

A string is a finite sequence of symbols from some alphabet.

Example :

" xyz " is a string over an alphabet $\Sigma = \{a, b, c, \ldots, z\}$. The empty string or null string is denoted by ϵ.

Length of a string

The length of a string is the number of symbols in that string. If w is a string then its length is denoted by $|w|$.

Example :

1. $w=abcd$, then length of w is $|w|= 4$

The set of strings of length K (K $\geq$ 1)

Let Σ be an alphabet and $\Sigma = \{a, b\}$, then all strings of length K ($K \geq 1$) is denoted by Σ^K.

$$\Sigma^K = \{w : w \text{ is a string of length } K, K \geq 1\}$$

Example:

1. $\Sigma = \{a,b\}$, then

   $\Sigma^1 = \{a,b\}$,

   $\Sigma^2 = \{aa, ab, ba, bb\}$,

   $\Sigma^3 = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

   $|\Sigma^1| = 2 = 2^1$ (Number of strings of length one),

Concatenation of strings

If w_1 and w_2 are two strings then concatenation of w_2 with w_1 is a string and it is denoted by w_1w_2. In other words, we can say that w_1 is followed by w_2 and $|w_1w_2| = |w_1| + |w_2|$.

Prefix of a string

A string obtained by removing zero or more trailing symbols is called prefix. For example, if a string $w = abc$, then a, ab, abc are prefixes of w.

Suffix of a string

A string obtained by removing zero or more leading symbols is called suffix. For example, if a string $w = abc$, then c, bc, abc are suffixes of w.
A string a is a proper prefix or suffix of a string w if and only if $a \neq w$.

Language

A Language L over Σ, is a subset of Σ^*, i. e., it is a collection of strings over the alphabet Σ. ϕ, and $\{\in\}$ are languages. The language ϕ is undefined as similar to infinity and $\{\in\}$ is similar to an empty box i.e. a language without any string.

Example:

1. $L_1 = \{01, 0011, 000111\}$ is a language over alphabet $\{0,1\}$
2. $L_2 = \{\in, 0, 00, 000, ...\}$ is a language over alphabet $\{0\}$
3. $L_3 = \{0^n 1^n 2^n : n \geq 1\}$ is a language.

Kleene Closure of a Language

Let L be a language over some alphabet Σ. Then Kleene closure of L is denoted by $L*$ and it is also known as reflexive transitive closure, and defined as follows :

$L^* = \{Set\ of\ all\ words\ over\ \Sigma\}$

$= \{$word of length zero, words of length one, words of length two, ...$\}$

$S = \{0\}$, then $S^* = \{\in, 0, 00, 000, 0000, 00000, ...\}$

Positive Closure

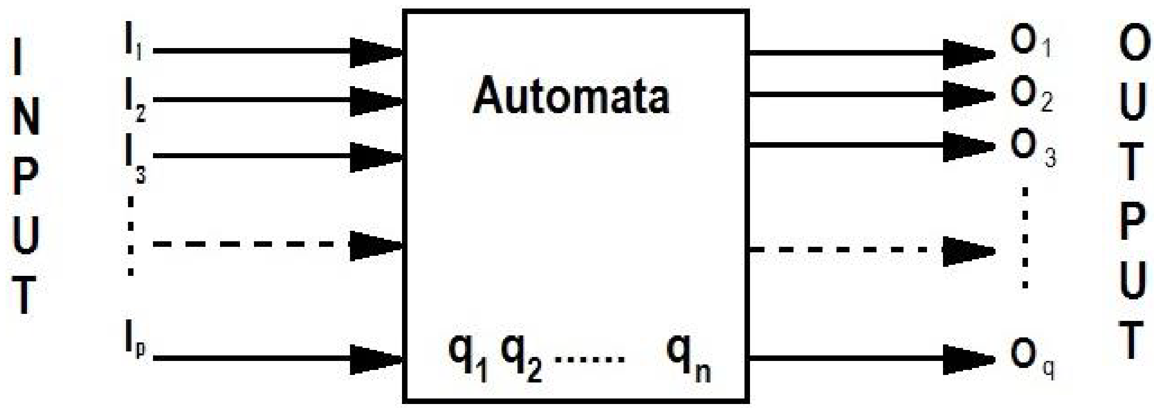
If Σ is an alphabet then positive closure of Σ is denoted by Σ^+ and defined as follows :

$\Sigma^+ = \Sigma^* - \{\in\} = \{Set\ of\ all\ words\ over\ \Sigma\ excluding\ empty\ string\ \in\}$

Example :

if $\Sigma = \{0\}$, then $\Sigma^+ = \{0, 00, 000, 0000, 00000, ...\}$

Model of Discrete Automata



Input: At each of the discrete instance of time t_1, t_2, t_3, ….., tn the input values are as I_1, I_2, I_3,….I_p, each of which can take a finite number of fixed values from the input alphabet Σ, are applied to the input side of the model.

Output: O_1, O_2, O_3, …., O_q, are the output of the discrete automata model, each of which can take a finite number of fixed values form an output O.

States: An state is an condition of processing the inputs. At any instant of time the automaton can be in one of the states q1, q2, q3, ….. qn.

State relation: The next state of an automaton at any instant of time is determined by the present state and the present input.

Output relation: The output is related to either state only or to both the input and the current state. It should be noted that at any instant of time the automaton is in some state. On reading an input symbol the automaton moves to a next state which is given by the state relation.